The Restricted Matched Filter for Distributed Detection

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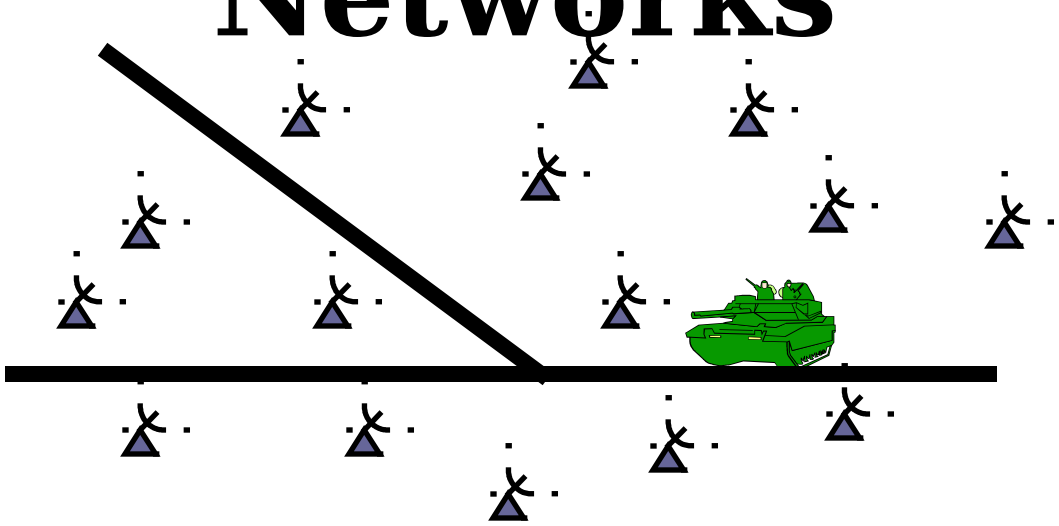
Outline

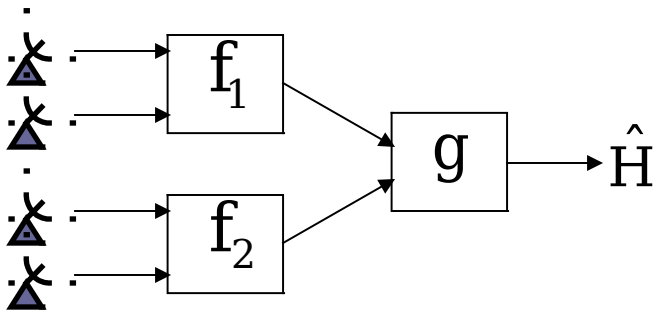
- **Distributed Detection Problem**
- **Motivation for the Restricted Matched Filter (RMF)**
- **Simulation Results**
- **Preliminary Conclusions**

Distributed Sensor Networks



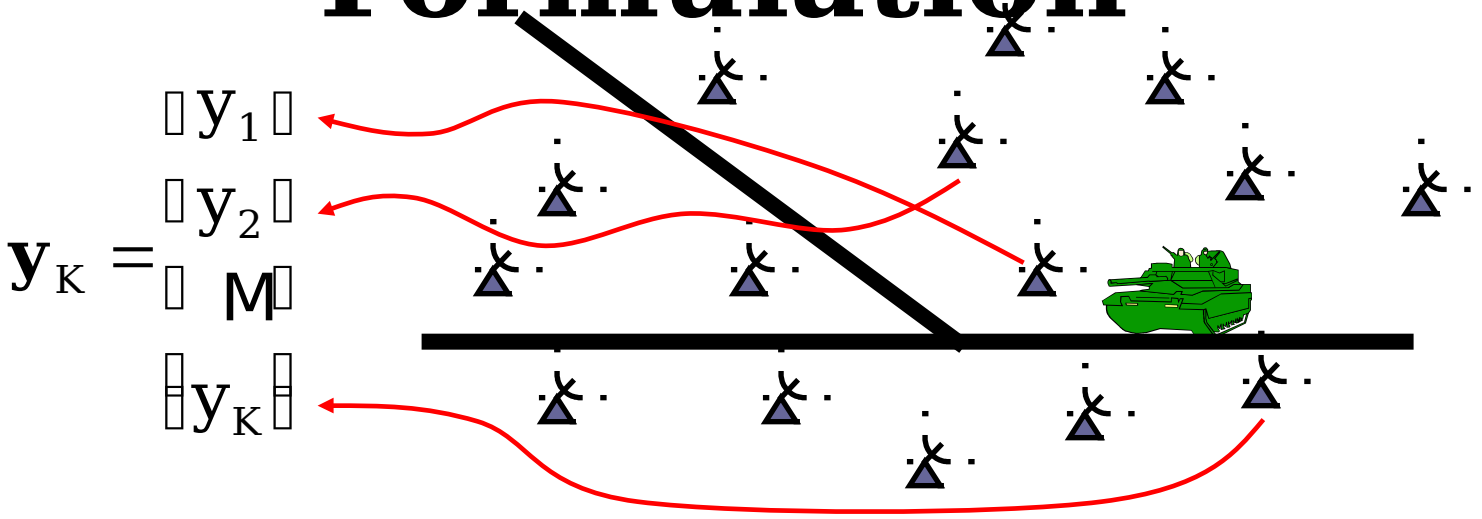
- **Detection algorithms incorporating all sensors produce high communication costs.**
- **Choosing a fixed number of sensor measurements for detection processing can reduce communication cost.**
- **RMF provides an upper bound to a sensor cluster's possible detection performance on an important class of signal models.**

General Distributed Detection Algorithms



- **Typically proposed algorithms combine quantized measurements from local sensor clusters.**
- **Design of these algorithms is complex. It involves search over algorithm topology and quantizer decision regions.**
- **Performance evaluation depends on algorithm topology.**
- **RMF offers a topology-independent way to upper bound the performance of a distributed detection algorithm.**

Detection Problem Formulation



- **Detection algorithm selects a fixed-size (K) subset of M sensors for best detection performance.**
- **Algorithm processes a snapshot of sensor measurements (values in y_K represent a spatial signal at a fixed time).**
- **No intermediate quantizers are included in the detector.**

Modeling Simplifications

- **Simplifying Assumptions:**
  - **In the presence of a target, the noise-free snapshot is known for all sensors.**
  - **Know noise correlation between sensors.**
  - **Gaussian noise.**
- **Formulate as a restricted matched filter problem.**

Notation

Target Absent: $\mathbf{y}_K = \mathbf{n}_K$

Target Present: $\mathbf{y}_K = \mathbf{s}_K + \mathbf{n}_K$

- $\mathbf{s}_K$ **is a known target signature.**
- $\mathbf{n}_K$ **is Gaussian noise with known covariance** Λ_K.
- **For any set of K sensors, the optimal detector is a matched filter. Sufficient statistic** l **is a linear function of the data.**

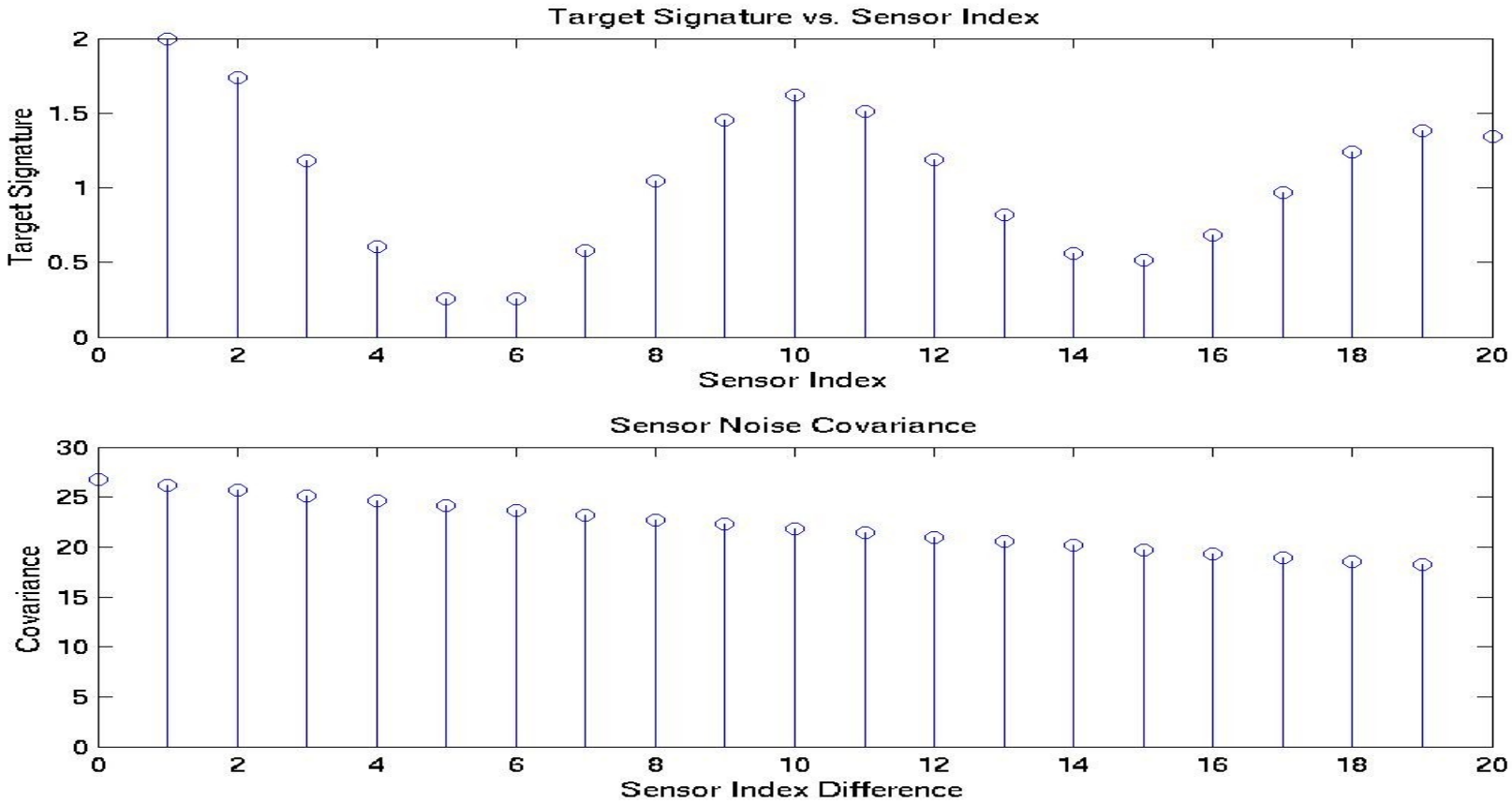
$$l = \mathbf{s}_K^T \Lambda_K^{-1} \mathbf{y}_K$$

- **Receiver operating characteristic (ROC) is determined by a single parameter.**

$$d_K^2 = \mathbf{s}_K^T \Lambda_K^{-1} \mathbf{s}_K$$

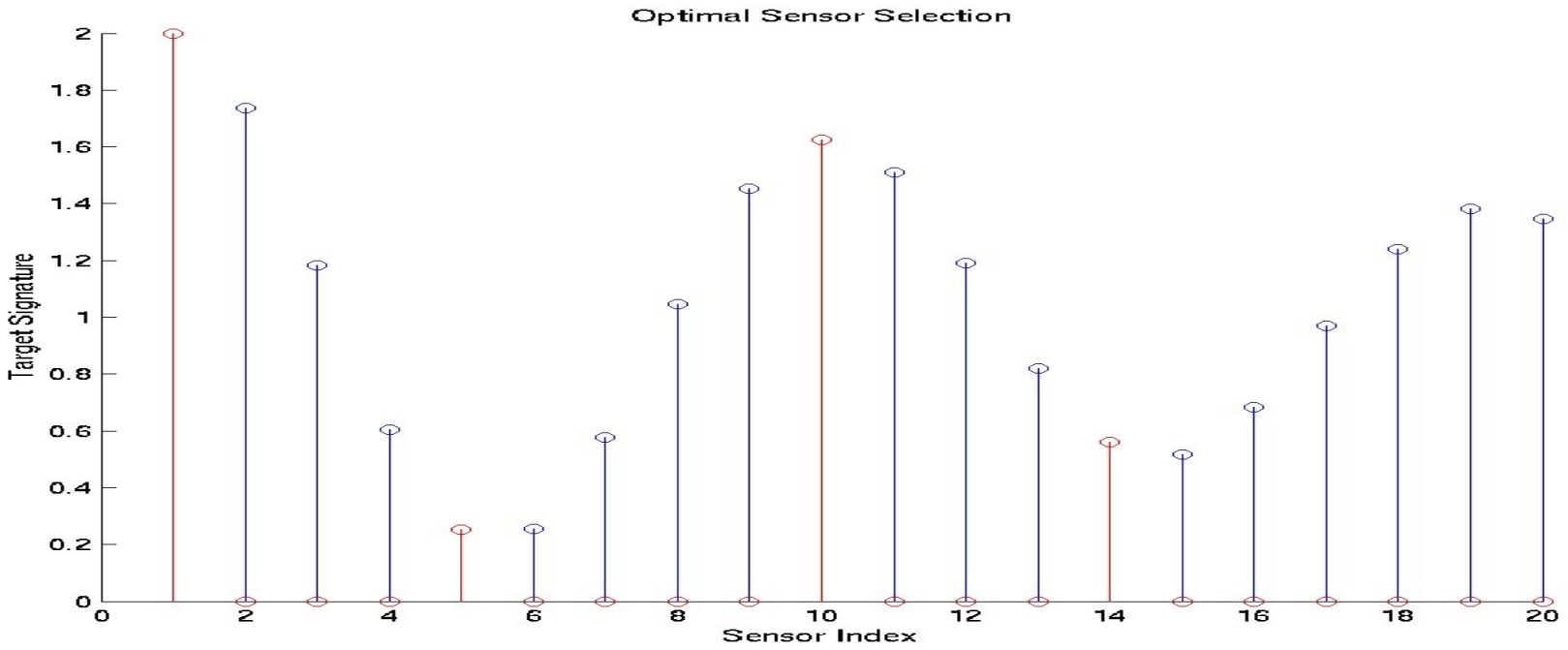
Example

- **Select subset of K = 4 sensors from a group of M = 20.**
- **Target signature and noise covariance are shown in figure.**



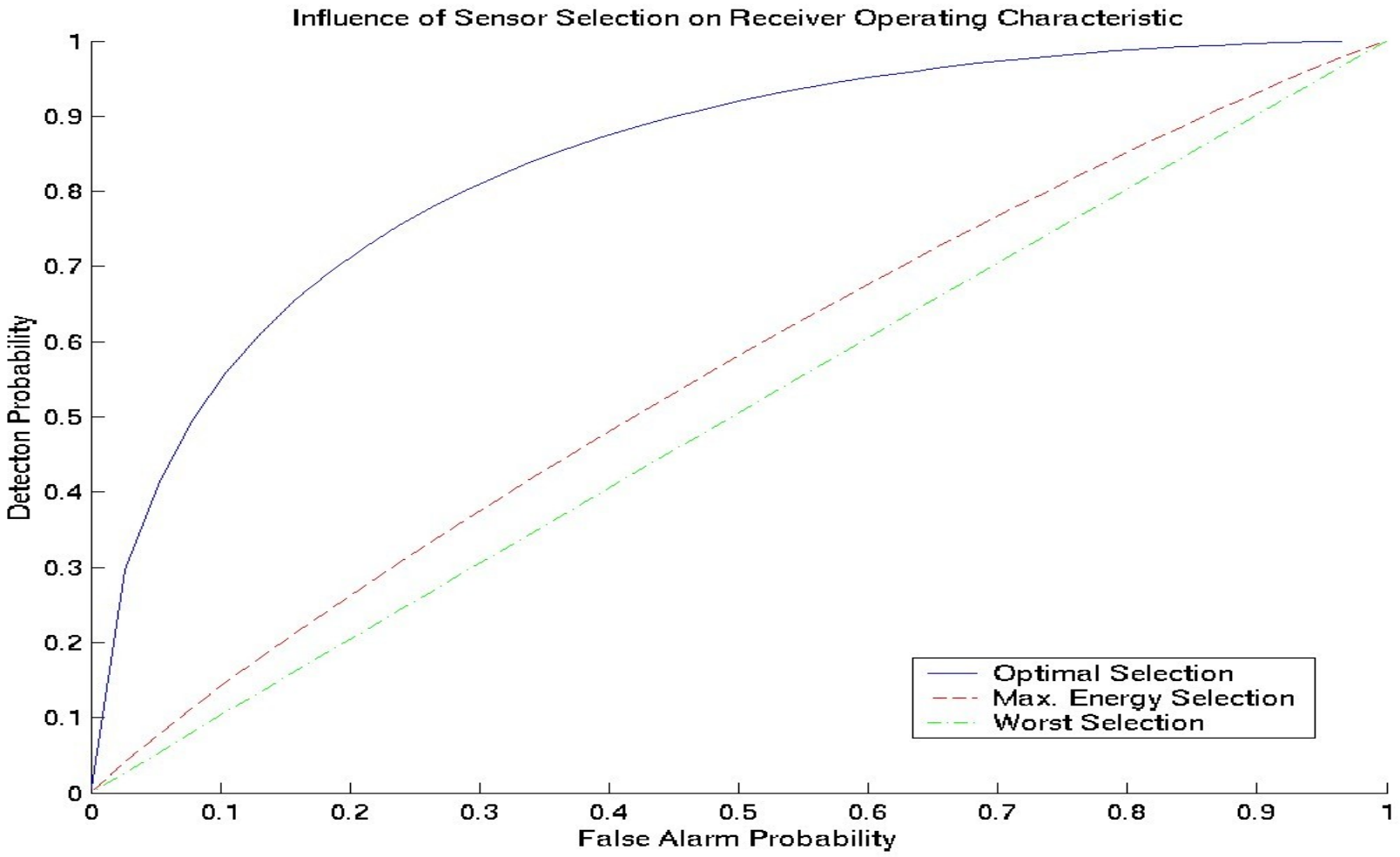
Tradeoff Between Signal Energy and Noise Correlation

- Generally, optimal subset does not have maximum energy in S_K.
  - Best subset balances energy in S_K and noise correlation.



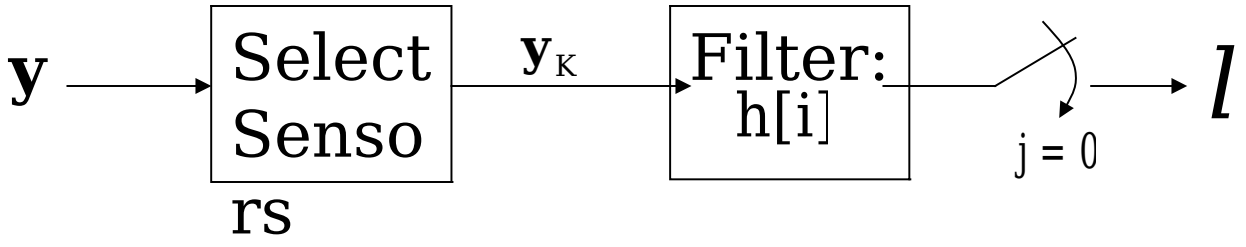
Importance of Sensor Choice

- **Figure shows ROCs for optimal RMF, maximum energy solution, and worst sensor selection.**



Restricted Matched Filter

- **For any K-sensor subset, the optimal detector is a matched filter.**
- **Performance depends upon intelligent selection of sensors.**
- **Qualitative analysis of RMF performance can improve efficiency of search algorithms.**



Qualitative Properties of Optimal Sensor Selection

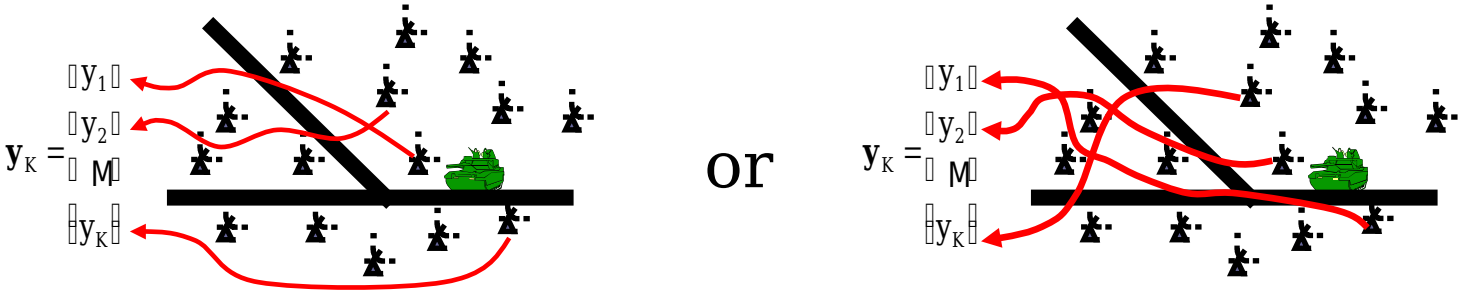
- **ROC is determined completely by a quadratic form. Eigenvalues and eigenvectors characterize performance.**

$$\Lambda_K = \mathbf{V}\Gamma\mathbf{V}^T \quad \Rightarrow \quad d_K^2 = \sum_{j=1}^{K} \frac{\mathbf{v}_j^T \mathbf{s}_K}{\gamma_j}$$

- **The optimal $\mathbf{s}_K$ occupies a subspace where noise is weak.**
  - **Optimal sensor selection steers the target signature into subspace spanned by eigenvectors associated with small eigenvalues.**
- **Qualitative characterization of the optimal sensor selection may improve the efficiency of search algorithms for the best RMF.**
  - **Search algorithms should optimize weighted projection of $\mathbf{s}_K$ onto eigenvectors of Λ_K.**

RMF Performance is Index Independent

- **RMF is a spatial filter, so data indexing is arbitrary.**



- **Optimal detector is linear. Rearrangement of data and filter coefficients does not affect sufficient statistic.**

$$l = \sum_{i=1}^{K} h[i]y[i$$

- **Index independence reduces complexity of search for optimal RMF.**

Bound Independent of Algorithm Topology

- **The RMF is the optimal detector for our hypothesis test.**
  - **Its ROC gives the maximum performance for any detector.**
  - **Implementation not specified by the form of the filter. The ROC depends only on sensor selection.**
- **Practical distributed detection algorithms can approximate the RMF if sufficient network bandwidth is available.**
  - **Weak quantization noise won't significantly affect the sufficient statistic .**

Conclusions

- **Optimal RMF gives an upper bound to distributed detection performance by a sensor cluster.**
- **RMF bound is independent of detection algorithm topology.**
- **Qualitative behavior of optimal RMF is determined by eigenvalues and eigenvectors of Λ_K.**
- **Current research issues:**
  - **Analytical results providing a qualitative characterization of optimal sensor selections.**
  - **Efficient search algorithms. Promises to produce practical detection algorithms if complexity is reduced sufficiently.**
  - **Application to more realistic data models reflecting uncertainty about target signature and sensor noise covariance.**